COMP4801 Project Plan
Research on Algorithm Design and Analysis for Bilateral Trade

Supervisor: Prof. Huang Zhiyi Student: Zhang Yiran

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Abstract

This project studies the bilateral trade model, where one buyer and one seller trade an indivisible good. Research shows that no truthful mechanisms can be socially efficient, many researchers have been working on truthful mechanisms that provide guaranteed approximations of the optimal efficiency. Generalized bilateral trade in two-sided markets has also been extensively studied. We will study the methods in related research papers and apply the approaches in our work on this model. We will also study the generalizations of bilateral trade that have multiple agents and multiple (weighted) items.

1 Introduction

This section provides relevant background information on the bilateral trade model, mechanism design, approximation algorithms, and the double auction model.

1.1 Bilateral Trade Model

In the bilateral trade model, a seller owns an indivisible item, and a buyer wants to purchase it. Both agents have private valuations about the item, we denote the seller's valuation as s and the buyer's valuation as b. We only know the public probability distribution (cumulative distribution functions) from which the valuations are drawn, i.e., $s \sim F_S$, $b \sim F_B$. We want the agent who values the item more to have the item, so a socially efficient mechanism always trades when the buyer has higher values, and never trades if the seller has higher values.

1.2 Mechanisms

A (possibly random) mechanism for bilateral trade is defined as $M = (x, p)$. The allocation function x is the probability of trade, and p specifies the price. We require that $s \leq p \leq b$, that is, both agents accept the price only if they expect non-negative payoffs. This constraint is due to individual rationality.

In some cases, agents may lie about their valuations in order to gain more benefits from trade. In incentive-compatible mechanisms, agents have no incentive to deviate

from using the strategy of their true valuations. By sticking to their true values, they have higher (expected) utilities.

A mechanism that does not rely on external subsidies to facilitate the trade is weakly budget balanced. If the payment of the buyer is fully transferred to the seller, the mechanism is strongly budget balanced.

In mechanism design, we pursue mechanisms that are individually rational, incentive-compatible, budget-balanced, and practical in implementation.

1.3 Social Welfare and Approximation Ratio

The social welfare is b if the trade occurs and s otherwise. The optimal social welfare is defined as $\text{OPT} = \mathbb{E}_{s\sim F_S, b\sim F_B}[\max\{b, s\}]$. The welfare of mechanism M is defined as $\text{ALG} = \mathbb{E}_{s\sim F_S, b\sim F_B}[b\cdot x + s\cdot(1-x)]$. Our work in this project focuses on the fixed-price mechanism. The mechanism posts a price p, and trade occurs if both agents accept the price. This mechanism is individually rational, incentive-compatible, and strongly budget-balanced. We can alternatively define $\text{ALG} = \mathbb{E}_{s\sim F_S, b\sim F_B}[b + (b-s)\cdot 1_{s\le p\le b}]$.

The approximation ratio describes the performance of the mechanism on the worst-case instance:

$$\max_{I=(F_S,F_B)} \frac{\text{OPT}(I)}{\text{ALG}(I)}$$

To minimize the above ratio is equivalent to maximize the welfare obtained by the mechanism.

1.4 Double Auction Model

The double auction setting is a natural generalization of the bilateral trade model. There are multiple sellers each holding an identical item, and multiple buyers each desiring one item. The valuations are drawn independently from an agent-specific distribution.

2 Related Research

In 1983, Myerson and Satterthwaite [8] initiated the study on the bilateral trade model. Their work shows that as long as the buyer and seller distributions are continuous with positive probability densities and the value distribution intervals overlap, no truthful mechanism can be socially efficient. Many researchers have been working on exploring what is possible.

Some fixed price mechanisms are efficient in approximating social welfare. The Median Mechanism [2] sets the median of the seller distribution as the price. This mechanism provides a 2 approximation. The Random Quantile Mechanism [3] chooses a quantile x randomly from a certain distribution and outputs the x-quantile of the seller distribution as the price. This mechanism is an $\frac{e}{e-1} \approx 1.58$ approximation. The Optimal Fixed Price Mechanism [4] [6] posts a price that maximizes the welfare of the mechanism. The approximation ratio is around 1.39, and in terms of using fixed price mechanisms to approximate welfare, this mechanism is the most efficient.

There is also much work on generalizations of bilateral trade on two-sided markets. The Trade Reduction Mechanism of McAfee [7] is asymptotically efficient in the double auction setting. This mechanism can be generalized and combined with the Seller/Buyer Offering Mechanism [1] and provide asymptotic efficiency in other generalized two-sided markets. Single sample algorithms combined with generalized VCG Mechanism [5] are also shown to be approximately efficient in generalized bilateral trade.

3 Methods

We will apply commonly used research methods in algorithm design and analysis in this project. We will also learn the techniques for specific problems in this area.

3.1 Theoretical Background Knowledge

- Algorithms
- Probability Theory
- Calculus and Analysis
- Optimization

3.2 Programming Languages

- Python
- C/C++
- Java

4 Objectives

4.1 In-depth study and improve current approaches

We study two papers on fixed price mechanisms [4] [6]. They respectively use a Quadratically Constrained Quadratic Program and a Dynamic Programming algorithm to numerically compute the guaranteed approximation ratio. They respectively show that fixed price mechanisms can get 0.72 and 0.71 fraction of the optimal welfare. And it is impossible to get 0.7381 fraction of the optimal welfare. They also state that the upper and lower bounds obtained using their methods converge with more computational resources. We will study the techniques used in their work and improve the efficiency and preciseness of the approaches. We will also apply our knowledge and try to gain more insights into this problem.

4.2 Work on generalized bilateral trade in two sided markets

We will study bilateral trade in generalized settings. The potential research problem settings include two sided markets with multiple agents, agents with multiple identical items, agents with multiple different items, etc.

5 Project Schedule

- Phase 1. Before Oct, 2024
  - In-depth study and improve current approaches.
- Phase 2. Oct, 2024 - Jan, 2025
  - Continue the work in Phase 1 and try to gain more insights.
- Phase 3. Jan, 2025 - Apr, 2025
  - Work on bilateral trade in generalized settings.

References

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